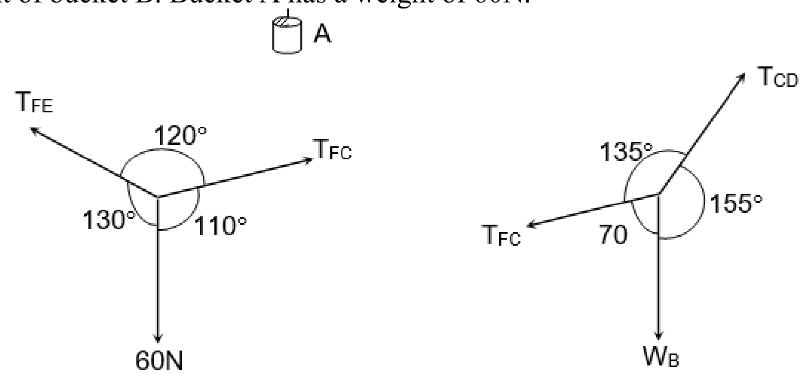
(Affiliated to University of Mumbai)	*End Semester Examination (R-24) SH 2025* *Answer Key with marking scheme*
Branch: All	Course: Applied Mechanics and Robot Dynamics
Year/ Semester: FY B.Tech / I	Course code: FEC105
Time: 02 hours	Marks: 60

		Marks
Q. 1	Attempt any **THREE.** (All questions carry equal marks)	**15**
A.	If the cords suspend the two buckets in the equilibrium position as shown in figure, determine the weight of bucket B. Bucket A has a weight of 60N.	05



Forces acting at point F — Forces acting at point C

Using Lami's theorem at point F.

$$\frac{60}{\sin 120} = \frac{T_{FE}}{\sin 110} = \frac{T_{FC}}{\sin 130}$$

Hence, T_{FE} = 65.1 N and T_{FC} = 53.07 N

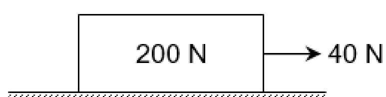
Using Lami's theorem at point C.

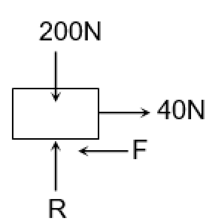
$$\frac{T_{FC}}{\sin 155^\circ} = \frac{W_B}{\sin 135^\circ}$$

Hence, W_B = 88.79 N

B.	A block of weight 200 N rests on a horizontal surface. The co–efficient of friction between the block and the horizontal surface is 0.4. Find the frictional force acting on the block if a horizontal force of 40 N is applied to the block	05

the block.





$\sum F_Y$ = 0

∴ R – 200 = 0

∴ R = 200 N

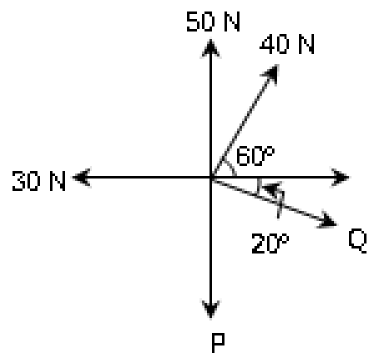
Maximum possible magnitude of frictional force = F_{max} = μR = 0.4 (200) = 80 N

But frictional force required for equilibrium, is F_{reqd} = 40 N

∴ F_{reqd} < F_{max}

∴ Motion is not impending and F = 40 N.

C. Five concurrent coplanar forces act on a body as shown in figure. Find the forces P and Q such that the resultant of the five forces is zero. **05**



$\sum F_x = 0 \quad \therefore \quad -30 + 40\cos 60° + Q\cos 20° = 0$

$\therefore \quad Q = 10.64 \text{ N}$

$\sum F_y = 0 \quad \therefore \quad 50 + 40\sin 60° - Q\sin 20° - P = 0$

$\therefore \quad P = 81 \text{ N}$

D. Equation of motion of a particle moving in a straight line is given by $s = 18t + 3t^2 - 2t^3$ where s is in metres and t is in seconds. Find: **05**

(i) Velocity and acceleration at start

(ii) Time when the particle reaches its max. velocity

(iii) Max. velocity of the particle.

$s = 18t + 3t^2 - 2t^3$ … (1)

$v = \frac{ds}{dt} = 18 + 6t - 6t^2$ … (2)

$a = \frac{dv}{dt} = 6 - 12t$ … (3)

(i) To find velocity at start, i.e. at t = 0,

$v_0 = 18$ m/s

$a_0 = 6$ m/s^2

(ii) We have seen in calculus that any function y reaches a local maximum when $y' = 0$ and $y'' < 0$.

For v to be maximum, $\frac{dv}{dt}$ must be zero i.e. a = 0 and $\frac{da}{dt} < 0$

∴ putting a = 0 in (3) we get

$0 = 6 - 12t$

∴ t = 0.5 seconds

Also, $\frac{da}{dt} = -12$

$\therefore \frac{da}{dt} < 0$

∴ at t = 0.5 seconds, velocity is maximum

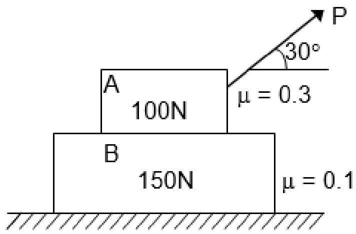
(iii) To find v_{max}

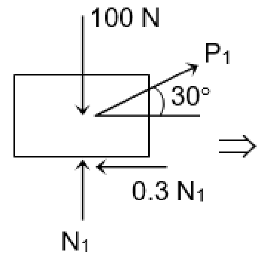
In equation (2), put t = 0.5 sec

$\therefore v_{max} = 18 + 6(0.5) - 6(0.5)^2$

$\therefore v_{max} = 19.5$ m/s

Q.2	Attempt any **THREE.** (All questions carry equal marks)	**30**
A.	I) Find out min. value of P to start the motion.	06





The motion can start in 2 ways

Case (i) Only A starts motion.

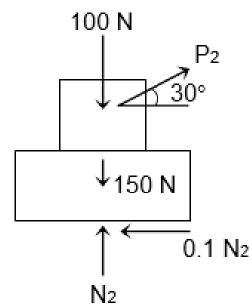
$\sum F_y = 0$ (↑ positive)

∴ $[P_1 \sin 30 + N_1 = 100]$

$\sum F_x = 0$ (→ positive)

∴ $[P_1 \cos 30 - 0.3 N_1 = 0]$

$P_1 = 29.52$ N & $N_1 = 85.23$ N



Case (ii) : Both A & B start motion together

$\sum F_y = 0$

$[P_2 \sin 30 + N_2 = 100 + 150]$

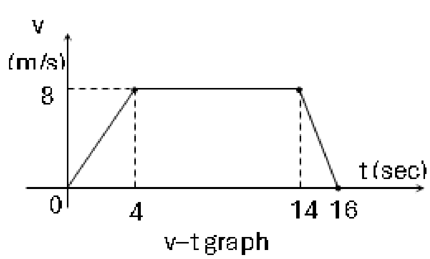
$\sum F_x = 0$

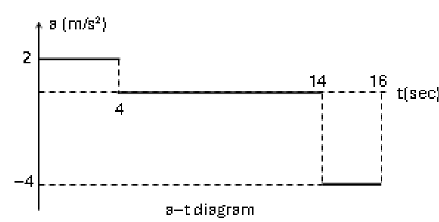
$[P_2 \cos 30 - 0.1 N_2 = 0]$

$P_2 = 27.29$ N & $N_2 = 236.35$ N

As $P_2 < P_1$, Case (ii) will take place at P = 27.29 N

II) The v–t diagram for a particle performing rectilinear translation is as shown. Draw a–t diagram for this motion. The particle starts from the origin x = 0. — 04





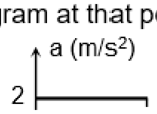
$$a_{4-14} = \frac{8-8}{14-4} = 0$$

Acceleration = slope of v–t diagram at that point

$$a_{16-14} = \frac{0-8}{16-14}$$

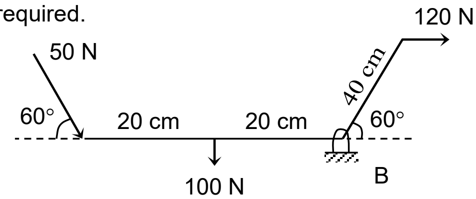
$$= -4 m/s^2$$

$$\therefore \quad a_{0-4} = \frac{8-0}{4-0} = 2 m/s^2$$



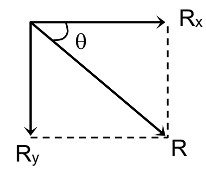
B. I) Find out resultant of given (lever) force system w.r.t. "B". **06**

(A) **Note :** Resultant is asked so don't show.
Support reactions i.e. FBD is not required.



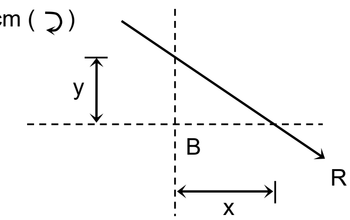
$R_x = \sum F_X$ (→ positive)
$= 50 \cos 60 + 120$
$= 145$ N(→)

$R_y = \sum F_y$ (↑ positive)
$= -50 \sin 60 - 100$
$= -143.3$
$= 143.3$ N (↓)



$R = \sqrt{R_x^2 + R_y^2} = 203.86$ N
$\theta = \tan^{-1}(R_y / R_x) = 44.66°$

(↺ positive) $\sum M_B = 50 \sin 60 \times 40 + 100 \times 20 - 120 \times 40 \sin 60$
$= -424.87 = 424.87$ Ncm (↻)



By Varignon's Principle

$x = \frac{\sum M}{R_y} = 2.96$ cm

$y = \frac{\sum M}{R_x} = 2.93$ cm

II) For a particle travelling along a linear path $a = t^2 + 1$ where a and t are in m/s2 and seconds respectively. Find the change in velocity between t = 3 sec and t = 6 sec. **04**

$a = t^2 + 1$

$\therefore \frac{dv}{dt} = t^2 + 1$

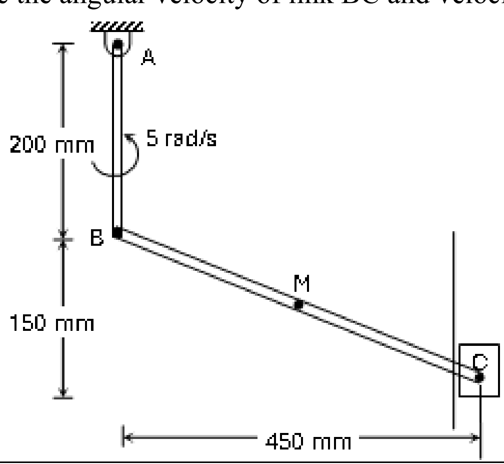
Let the velocities at t = 3 sec. and t = 6 sec. be v_3 and v_6 respectively.

$\therefore \int_{v_3}^{v_6} dv = \int_3^6 (t^2 + 1) dt$

$\therefore v_6 - v_3 = \left[\frac{t^3}{3} + t\right]_3^6 = 78 - 12 = 66$ m/s

Hence change in velocity in 66 m/s

C. I) In the mechanism shown the angular velocity of link AB is 5 rad/sec anticlockwise. At the instant shown, determine the angular velocity of link BC and velocity of piston C. **06**



For Rod AB : $V_B = l(I_1B) \times \omega_1$

$= 0.2 \times 5$

$= 1$ m/s (→)

For Rod BC : $\omega_2 = \dfrac{V_B}{l(I_2B)}$

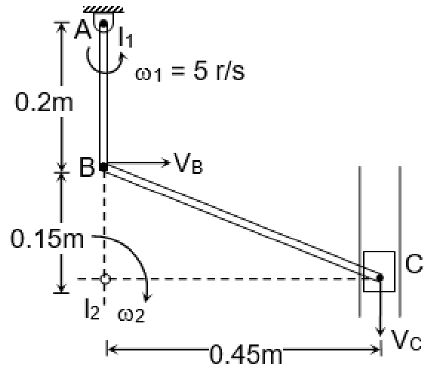
$= \dfrac{1}{0.15}$

$\omega_2 = 6.67$ r/s (↻)

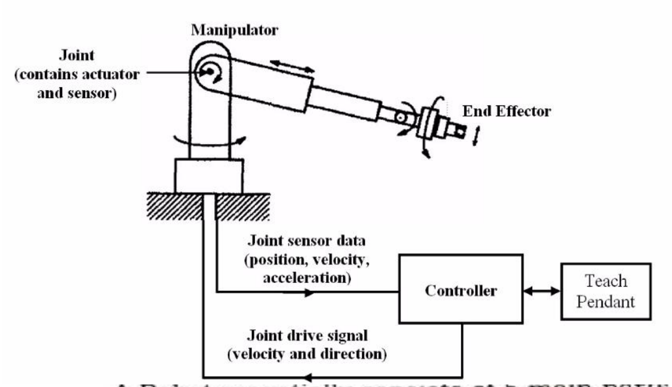
and $V_C = l(I_2V) \times \omega_2$

$= 0.45 \times 6.67$

$V_C = 3$ m/s (↓)



II) Explain with Neat sketch main parts of Robots 04



A Robot essentially consists of 5 main parts

1. **Manipulator** - It makes up the main structure of the robot. It consists of links and joints and gives the shape and form to a robot.

   Joints in a robot are mainly of two kinds viz. Revolute and Prismatic. A revolute joint allows only rotation and does not allow any linear motion at the joint. Whereas a Prismatic joint allows linear motion at the joint and does not facilitate any rotation.

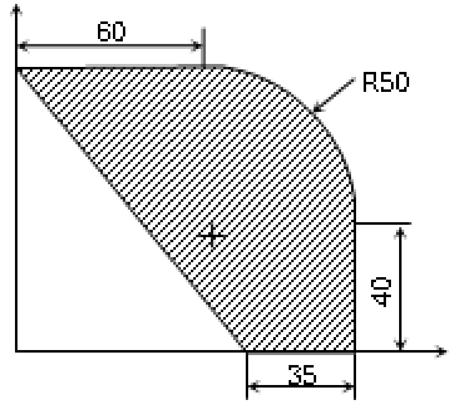
2. **End effector** - It is the last extreme part of a robot. It is designed to interact with the environment. It is finally the end effector which performs the task for which the robot is designed. The type of end effector depends on the task to be performed. For example, for a robot designed for picking an object from one point and placing at some other point, the end effector would be a gripper, which should hold and also release the object as desired.

3. **Actuators** - Actuators are devices which provide motion to the joints and links. Actuators convert either electrical, air or hydraulic energy into mechanical energy. Actuators may be servo motors or pneumatic actuators or hydraulic actuators.

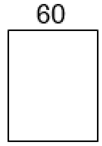
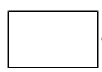
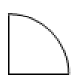
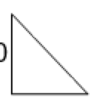
4. **Sensors** - Sensors monitor the robot's internal and external environment. Sensors track the position, orientation, speed, acceleration, and other changes and sends the same as input signal to the controller for adjusting robot's movement.

5. **Controller** - It is the brain of a robot. It receives command inputs from the computer and signals from the sensors. On these bases it directs and controls the different actuators placed at different joints for the designed motion.

D. I) Determine Centroid of the shaded area **08**



Assuming all in (mm)

Parts	A(mm^2)	x(mm)	y(mm)
60  90	60×90	$\frac{60}{2} = 30$	$\frac{90}{2} = 45$
+  40, 50	50×40	$60 + \frac{50}{2} = 85$	$\frac{40}{2} = 20$
+  R50	$\frac{\pi(50)^2}{4}$	$60 + \frac{4(50)}{3\pi}$	$40 + \frac{4(50)}{3\pi}$
– 90  75	$-\frac{75 \times 90}{2}$	$\frac{75}{3} = 25$	$\frac{90}{3} = 30$
Σ	5988.5	–	–

$$\bar{x} = \frac{\Sigma(Ax)}{\Sigma(A)} = \frac{407101.4}{5988.5} = 67.98 \text{ mm}$$

$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma(A)} = \frac{301956.5}{5988.5} = 50.42 \text{ mm}$$

II) Define Angle of Repose and Angle of Friction **02**

- **Angle of Repose (θ)**

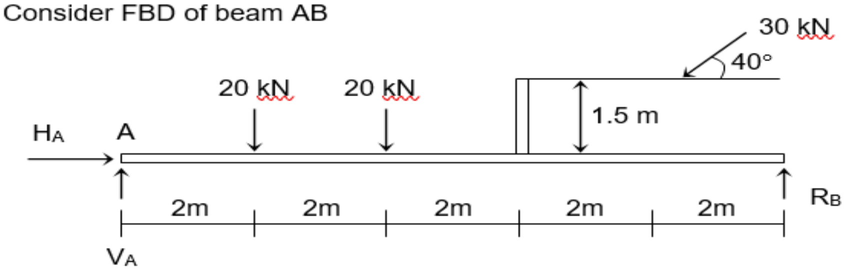
If a block is placed on a rough inclined plane and if the inclination of the plane is gradually increased, then the angle θ at which the block would have impending motion down the slope is called the angle of repose.

- **Angle of Friction**

The angle made by the resultant R with the normal to the surface of contact when the body has impending motion is called the angle of friction.

E. Find the support reactions at Hinge A and Roller B. **10**

Consider FBD of beam AB



$\Sigma M_A = 0$ (↺ positive)

$\therefore\ -20 \times 2 - 25 \times 4 - 30 \sin 40 \times 8 + 30 \cos 40 \times 1.5 + R_B \times 10 = 0$

$R_B = 25.98$ kN (↑)

$\Sigma F_x = 0$ (→ positive)

$\therefore\ H_A - 30 \cos 40 = 0$

$\therefore\ H_A = 22.98$ kN (→)

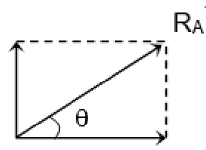
$\Sigma F_y = 0$ (↑ positive)

$\therefore\ V_A - 20 - 25 - 30 \sin 40 + R_B = 0$

$V_A = 38.30$ kN (↑)

$$R_A = \sqrt{H_A^2 + V_A^2} = 44.67 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{V_A}{H_A}\right) = 59.04°$$



Q.3	Attempt any **THREE.** (All questions carry equal marks)	**15**
A.	Write a Homogeneous matrix that represents Pure Rotation about all the 3 axes	05

Projection of z_1 on $z = 1$

Putting this vector data in matrices form, we have, Rotation Matrix about z axis as $R_{OT}(z)$

$$R_{OT}(z) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (6)$$

Similarly, Rotation Matrix about x axis can be obtained as $R_{OT}(x)$

$$R_{OT}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (7)$$

Similarly, Rotation Matrix about y axis can be obtained as $R_{OT}(y)$

$$R_{OT}(y) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots (8)$$

B.	Rod AB of length 3 m is kept on smooth planes as shown in figure. The velocity of end A is 5 m/s along the inclined plane. Locate the ICR and find the velocity of end B.	05

Using sine Rule in ΔAIB

$$\frac{3}{\sin 30^\circ} = \frac{IA}{\sin 70^\circ} = \frac{IB}{\sin 80^\circ}$$

$\therefore$ IA = 5.638 m

$\therefore$ IB = 5.909 m

V_A = IA $\times$ ω_{AB} ; I is the ICR

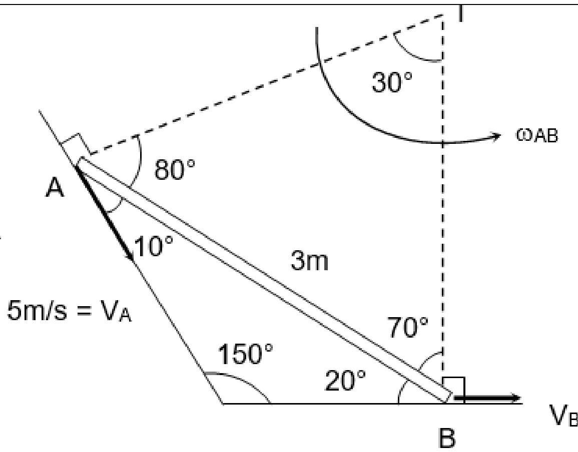
5 = 5.638 $\times$ ω_{AB}

$\therefore$ ω_{AB} = 0.8868 r/s (↻)

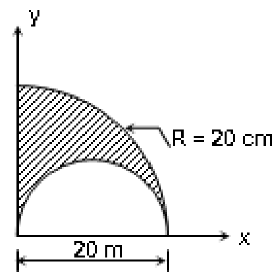
V_B = IB $\times$ ω_{AB}

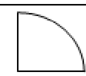
= 5.909 $\times$ 0.8868

= 5.24 m/s (→)



C. Find the centroid for shaded area. **05**



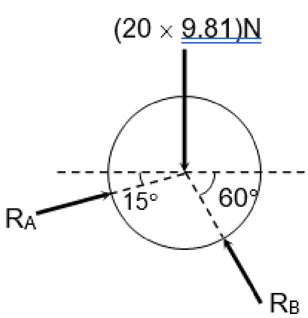
Part	A(cm²)	x(cm)	y(cm)
 R = 20	$\frac{\pi(20)^2}{4}$	$\frac{4(20)}{3\pi}$	$\frac{4(20)}{3\pi}$
– 	$\frac{-\pi(10)^2}{2}$	10	$\frac{4(10)}{3\pi}$

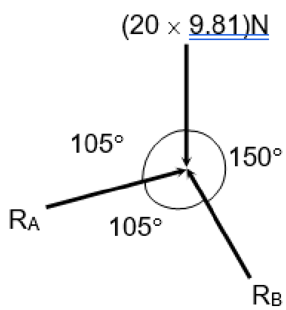
$$\bar{x} = \frac{\sum(Ax)}{\sum(A)} = \frac{2666.67 - 1570.8}{157.08} = 6.98 \text{ cm}$$

$$\bar{y} = \frac{\sum(Ay)}{\sum(A)} = \frac{2666.67 - 666.67}{157.08} = 12.73 \text{ cm}$$

Centroid G $\equiv$ $(\bar{x}, \bar{y})$ = (6.98, 12.73) cm

D. A sphere of mass 20 kg is resting against two inclined planes with inclinations of 75° and 30° respectively. Calculate the contact forces (reactions) at A and B. **05**





FBD of sphere

By Lami's theorem,

$$\frac{20 \times 9.81}{\sin 105^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{R_B}{\sin 105^\circ}$$

$\therefore$ R_A = 101.6 N, R_B = 196.2 N

********* **All the Best** *********